

$$K_L \rightarrow \pi^+ \pi^- e^+ e^- *$$

John K. Elwood and Mark B. Wise

*California Institute of Technology, Pasadena, CA 91125*

*and*

Martin J. Savage

*Department of Physics, Carnegie Mellon University, Pittsburgh PA 15213*

### Abstract

We calculate all of the form factors for the one-photon,  $K_L \rightarrow \pi^+ \pi^- \gamma^* \rightarrow \pi^+ \pi^- e^+ e^-$  contribution to the  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  decay amplitude at leading order in chiral perturbation theory. These form factors depend on one unknown constant that is a linear combination of coefficients of local  $\mathcal{O}(p^4)$  operators in the chiral lagrangian for weak radiative kaon decay. We determine the differential rate for  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  and also the magnitude of two CP violating observables.

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## 1. Introduction

At the present time about twenty  $K_L \rightarrow \pi^+\pi^-e^+e^-$  events have been observed and a detailed experimental study of this decay mode will be possible in future experiments [1]. The  $K_L \rightarrow \pi^+\pi^-e^+e^-$  weak decay amplitude is dominated by the process,  $K_L \rightarrow \pi^+\pi^-\gamma^* \rightarrow \pi^+\pi^-e^+e^-$ , where a single virtual photon creates the  $e^+e^-$  pair. This one photon contribution to the decay amplitude has the form

$$M^{(1\gamma)} = \frac{s_1 G_F \alpha}{4\pi f q^2} \left[ i G \varepsilon^{\mu\lambda\rho\sigma} p_{+\lambda} p_{-\rho} q_\sigma + F_+ p_+^\mu + F_- p_-^\mu \right] \cdot \bar{u}(k_-) \gamma_\mu v(k_+) \quad , \quad (1.1)$$

where  $G_F$  is Fermi's constant,  $\alpha$  is the electromagnetic fine structure constant,  $s_1 \simeq 0.22$  is the sine of the Cabibbo angle and  $f \simeq 132$  MeV is the pion decay constant. The  $\pi^+$  and  $\pi^-$  four-momenta are denoted by  $p_+$  and  $p_-$  and the  $e^+$  and  $e^-$  four-momenta are denoted by  $k_+$  and  $k_-$ . The sum of the electron and positron four-momenta is  $q = k_- + k_+$ . The Lorentz scalar form factors  $G, F_\pm$  depend on the scalar products of the four-momenta  $q, p_+$  and  $p_-$ . Neglecting CP nonconservation, under interchange of the pion four-momenta

$$p_+ \rightarrow p_- \quad \text{and} \quad p_- \rightarrow p_+ \quad (1.2)$$

the form factors become

$$G \rightarrow G \quad , \quad F_+ \rightarrow F_- \quad , \quad F_- \rightarrow F_+ \quad . \quad (1.3)$$

In this paper we compute the CP conserving contribution to the form factors  $G, F_\pm$  using chiral perturbation theory at one-loop order (the  $\mathcal{O}(p^2)$  amplitude vanishes). The coefficients of some of the local operators appearing at the same order in the chiral expansion (i.e., order  $p^4$  counter terms, where  $p$  is a typical momentum) are determined by the experimental value of the pion charge radius and the measured  $K^+ \rightarrow \pi^+e^+e^-$  and  $K_L \rightarrow \pi^+\pi^-\gamma$  decay rates and spectra.

We also compute (in chiral perturbation theory) an important tree level contribution to the form factors  $F_\pm$  that arises from the small CP even component of the  $K_L$  state. This contribution to the  $F_\pm$  form factors from indirect CP nonconservation has the opposite symmetry property under interchange of pion momenta when compared with the CP conserving contribution to  $F_\pm$  (see eqs. (1.2) and (1.3)) . If

$$p_+ \rightarrow p_- \quad \text{and} \quad p_- \rightarrow p_+ \quad (1.4)$$

then the CP violating one-photon form factors become

$$F_+ \rightarrow -F_- \quad , \quad F_- \rightarrow -F_+ \quad . \quad (1.5)$$

The decay amplitude that follows from squaring the invariant matrix element in eq. (1.1) and summing over  $e^+$  and  $e^-$  spins is symmetric under interchange of  $e^+$  and  $e^-$  momenta,  $k_- \leftrightarrow k_+$ . Physical variables that are antisymmetric under interchange of the  $e^+$  and  $e^-$  momenta arise from the interference of the short distance contributions ( $Z$ -penguin and  $W$ -box diagrams) and the two photon piece with the one photon amplitude given in eq. (1.1).

In the minimal standard model the coupling of the quarks to the  $W$ -bosons has the form

$$\mathcal{L}_{int} = -\frac{g_2}{\sqrt{2}} \bar{u}_L^j \gamma_\mu V^{jk} d_L^k W^\mu + h.c. \quad . \quad (1.6)$$

Here repeated generation indices  $j, k$  are summed over 1,2,3 and  $g_2$  is the weak SU(2) gauge coupling.  $V$  is a  $3 \times 3$  unitary matrix (the Cabibbo–Kobayashi–Maskawa matrix) that arises from diagonalization of the quark mass matrices. By redefining phases of the quark fields it is possible to write  $V$  in terms of four angles  $\theta_1, \theta_2, \theta_3$  and  $\delta$ . The  $\theta_j$  are analogous to the Euler angles and  $\delta$  is a phase that, in the minimal standard model, is responsible for the observed CP violation. Explicitly

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \quad , \quad (1.7)$$

where  $c_i \equiv \cos \theta_i$  and  $s_i \equiv \sin \theta_i$ . It is possible to choose the  $\theta_j$  to lie in the first quadrant. Then the quadrant of  $\delta$  has physical significance and cannot be chosen by a phase convention for the quark fields. A value of  $\delta$  not equal to 0 or  $\pi$  gives rise to CP violation.

The short distance  $W$ -box and  $Z$ -penguin Feynman diagrams depend on the  $V_{ts}$  element of the Cabibbo–Kobayashi–Maskawa matrix. It is very important to be able to determine this coupling experimentally. In this paper we calculate the contribution to the  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  decay amplitude arising from the  $Z$ -penguin and  $W$ -box diagrams which can be determined using chiral perturbation theory since the left handed current  $\bar{s} \gamma_\mu (1 - \gamma_5) d$  is related to a generator of chiral symmetry. At the present time all observed CP nonconservation has its origin in  $K^0 - \bar{K}^0$  mass mixing. A CP violating variable can be constructed in the decay  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  that gets an important contribution from

CP nonconservation in the  $Z$ -penguin and  $W$ -box diagrams, that is, direct CP violation. The variable (in the  $K_L$  rest frame)

$$A_{CP} = \frac{(\vec{p}_- \times \vec{p}_+) \cdot (\vec{k}_- - \vec{k}_+)}{|(\vec{p}_- \times \vec{p}_+) \cdot (\vec{k}_- - \vec{k}_+)|} > , \quad (1.8)$$

is even under charge conjugation and odd under parity. It is also odd under interchange of  $\vec{k}_+$  and  $\vec{k}_-$ . The real and imaginary parts of  $V_{ts}$  are comparable, and hence the CP conserving and CP violating parts of the  $Z$ -penguin and  $W$ -box diagrams are of roughly equal importance.  $A_{CP}$  gets a significant contribution from this direct source of CP nonconservation. In this paper we calculate  $A_{CP}$  in the minimal standard model but unfortunately we find that it is quite small;  $|A_{CP}| \approx 10^{-4}$ .

The decay  $K_L \rightarrow \pi^+\pi^-e^+e^-$  has been studied previously by Sehgal and Waininger [2] and by Heiliger and Sehgal [3]. These authors adopted a phenomenological approach, relating the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  decay amplitude to the measured  $K_L \rightarrow \pi^+\pi^-\gamma$  decay amplitude. In the systematic expansion of chiral perturbation theory we find important additional contributions to the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  decay amplitude for  $q^2 = (k_- + k_+)^2 \gg 4m_e^2$  that were not included in this previous work. It was pointed out in refs. [2] and [3] that indirect CP nonconservation from  $K^0 - \bar{K}^0$  mixing gives an important contribution to the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  decay rate and consequently there is a CP violating observable,  $B_{CP}$ , that is quite large. We reexamine  $B_{CP}$  using the form factors determined in this paper.

## 2. The One-Photon Amplitude

Chiral perturbation theory provides a systematic approach to understanding the one-photon part of the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  decay amplitude. It uses an effective field theory that incorporates the  $SU(3)_L \times SU(3)_R$  chiral symmetry of QCD and an expansion in powers of momentum to reduce the number of operators that occur. In the chiral Lagrangian the  $\pi$ 's,  $K$ 's and  $\eta$  are incorporated into a  $3 \times 3$  special unitary matrix

$$\Sigma = \exp \left( \frac{2iM}{f} \right) , \quad (2.1)$$

where

$$M = \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} . \quad (2.2)$$

At leading order in chiral perturbation theory  $f \simeq 132$  MeV is the pion decay constant. Under  $SU(3)_L \times SU(3)_R$  transformations the  $\Sigma$  field transforms as

$$\Sigma \rightarrow L \Sigma R^\dagger \quad , \quad (2.3)$$

where  $L \in SU(3)_L$  and  $R \in SU(3)_R$ .

At leading order in chiral perturbation theory (i.e., order  $p^2$ , where  $p$  is a typical four-momentum) the strong and electromagnetic interactions of the pseudo-Goldstone bosons are described by the chiral Lagrange density

$$\mathcal{L}_S^{(1)} = \frac{f^2}{8} \text{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger) + v \text{Tr}(m_q \Sigma + m_q \Sigma^\dagger) \quad , \quad (2.4)$$

where  $v$  is a parameter with dimensions of mass to the third power and  $m_q$  is the quark mass matrix

$$m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \quad . \quad (2.5)$$

In this paper we neglect isospin violation in the quark mass matrix and set  $m_u = m_d$ . In this approximation the  $K^0$  and  $K^+$  have equal masses which we denote by  $m_K$ , and the Gell-Mann–Okubo mass relation

$$3m_\eta^2 - 4m_K^2 + m_\pi^2 = 0 \quad , \quad (2.6)$$

holds.

The effective Lagrangian for  $\Delta S = 1$  weak nonleptonic decays transforms as  $(8_L, 1_R) + (27_L, 1_R)$  under  $SU(3)_L \otimes SU(3)_R$ . The  $(8_L, 1_R)$  amplitudes are much larger than the  $(27_L, 1_R)$  amplitudes and so we will neglect the  $(27_L, 1_R)$  part of the effective Lagrangian. The effective Lagrangian for weak radiative kaon decay is obtained by gauging the effective Lagrangian for weak nonleptonic decays with respect to the  $U(1)_Q$  of electromagnetism. At leading order in chiral perturbation theory the  $\Delta S = 1$  transitions are described by

$$\mathcal{L}_W^{(1)} = \frac{g_8 G_F s_1 f^4}{4\sqrt{2}} \text{Tr} [D_\mu \Sigma D^\mu \Sigma^\dagger T] + h.c. \quad . \quad (2.7)$$

The matrix  $T$  in (2.7) projects out the correct flavour structure of the octet

$$T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad , \quad (2.8)$$

and  $g_8$  is a constant determined by the measured  $K_S \rightarrow \pi^+\pi^-$  decay rate;  $|g_8| \simeq 5.1$ . In (2.4) and (2.7)  $D_\mu$  represents a covariant derivative:

$$D_\mu \Sigma = \partial_\mu \Sigma + ieA_\mu [Q, \Sigma] \quad , \quad (2.9)$$

where

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} \quad , \quad (2.10)$$

is the electromagnetic charge matrix for the three lightest quarks,  $u, d$  and  $s$ .

The  $K_L$  state

$$|K_L\rangle \simeq |K_2\rangle + \epsilon |K_1\rangle \quad , \quad (2.11)$$

is mostly the CP odd state

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \quad , \quad (2.12)$$

with small admixture of the CP even state

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \quad . \quad (2.13)$$

The parameter  $\epsilon$  characterizes CP nonconservation in  $K^0 - \bar{K}^0$  mixing. At leading order in chiral perturbation theory the  $K_L \rightarrow \pi^+ \pi^- \gamma^* \rightarrow \pi^+ \pi^- e^+ e^-$  decay amplitude arises through the CP even component of  $K_L$ . Writing the form factors contributing to  $K_L \rightarrow \pi^+ \pi^- \gamma^*$  as a power series in the chiral expansion

$$F_\pm = F_\pm^{(1)} + F_\pm^{(2)} + \dots \quad , \quad G = G^{(1)} + G^{(2)} + \dots \quad , \quad (2.14)$$

where the superscript denotes the order of chiral perturbation theory, we find that the Feynman diagrams in fig. 1 give

$$\begin{aligned} G^{(1)} &= 0 \\ F_+^{(1)} &= -\frac{32g_8 f^2 (m_K^2 - m_\pi^2) \pi^2 \epsilon}{[q^2 + 2q \cdot p_+]} \quad . \\ F_-^{(1)} &= +\frac{32g_8 f^2 (m_K^2 - m_\pi^2) \pi^2 \epsilon}{[q^2 + 2q \cdot p_-]} \end{aligned} \quad (2.15)$$

Despite the fact that  $\epsilon \simeq 0.0023 e^{i44^\circ}$  (in a phase convention where the  $K^0 \rightarrow \pi\pi(I=0)$  decay amplitude is real), is small it is important to keep this part of the decay amplitude.

Other contributions not proportional to  $\epsilon$  don't occur until higher order in chiral perturbation theory. We neglect direct sources of CP nonconservation in the one-photon part of the decay amplitude. Experimental information on  $\epsilon'$  suggests that they are small.

At the next order in the chiral expansion the form factors  $G^{(2)}, F_{\pm}^{(2)}$  arise from  $\mathcal{O}(p^4)$  local operators and from one-loop Feynman diagrams involving vertices from the leading Lagrange densities in (2.4) and (2.7). However, the form factor  $G^{(2)}$  arises solely from local operators as the one loop Feynman diagrams and tree graphs involving the Wess–Zumino term [4] [5] do not contribute. The contribution of the  $\mathcal{O}(p^4)$  local operators to  $G^{(2)}$  is fixed by the measured  $K_L \rightarrow \pi^+\pi^-\gamma$  decay rate [6] [7] to be

$$|G^{(2)}| \simeq 40 \quad . \quad (2.16)$$

The experimentally observed  $K_L \rightarrow \pi^+\pi^-\gamma$  Dalitz plot suggests that the form factor  $G$  has significant momentum dependence. This indicates that  $G^{(3)}$  is not negligible, and that our extraction of  $G^{(2)}$  from the rate is not completely justified [8].

The form-factors  $F_{\pm}^{(2)}$  get contributions both from local operators of  $\mathcal{O}(p^4)$  [9] and from one-loop diagrams involving vertices from the leading Lagrange densities in (2.4) and (2.7). For  $K_L \rightarrow \pi^+\pi^-e^+e^-$  the local operators that contribute are

$$\mathcal{L}_S^{(2)} = \frac{-ie\lambda_{cr}(\mu)}{16\pi^2} F^{\mu\nu} Tr [Q(D_\mu \Sigma D_\nu \Sigma^\dagger + D_\mu \Sigma^\dagger D_\nu \Sigma)] \quad , \quad (2.17)$$

and

$$\begin{aligned} \mathcal{L}_W^{(2)} = i \frac{G_F s_1 e f^2 g_8}{\sqrt{2} 16\pi^2} [ & a_1(\mu) F^{\mu\nu} Tr [QT(\Sigma D_\mu \Sigma^\dagger)(\Sigma D_\nu \Sigma^\dagger)] \\ & + a_2(\mu) F^{\mu\nu} Tr [Q(\Sigma D_\mu \Sigma^\dagger)T(\Sigma D_\nu \Sigma^\dagger)] \\ & + a_3(\mu) F^{\mu\nu} Tr [T[Q, \Sigma]D_\mu \Sigma^\dagger \Sigma D_\nu \Sigma^\dagger - TD_\mu \Sigma D_\nu \Sigma^\dagger \Sigma[\Sigma^\dagger, Q]] \\ & + a_4(\mu) F^{\mu\nu} Tr [T\Sigma D_\mu \Sigma^\dagger [Q, \Sigma] D_\nu \Sigma^\dagger] ] + h.c. \end{aligned} \quad (2.18)$$

The coefficients  $\lambda_{cr}, a_1, a_2, a_3$  and  $a_4$  depend on the renormalization procedure used and we employ dimensional regularization with  $\overline{MS}$  subtraction. The dependence of the coefficients  $\lambda_{cr}, a_{1,2,3,4}$  on the subtraction point  $\mu$  cancels that coming from the one-loop diagrams. Note that the basis of operators in eq. (2.18) is slightly different than that used in [9]. With this basis of operators the combination of counterterms

$$w_L = a_3 - a_4 \quad , \quad (2.19)$$

is independent of the subtraction point  $\mu$  at one loop.

The value of  $\lambda_{cr}$  is fixed by the measured  $\pi^+$  charge radius;  $\langle r_\pi^2 \rangle = 0.44 \pm 0.02 \text{ fm}^2$ . The one-loop diagrams in fig. 2 give (using  $\overline{MS}$ )

$$\lambda_{cr}(\mu) = - \left( \frac{2\pi^2}{3} \right) f^2 \langle r_\pi^2 \rangle - \frac{1}{24} [2 \ln(m_\pi^2/\mu^2) + \ln(m_K^2/\mu^2)] \quad , \quad (2.20)$$

which implies that (at the subtraction point  $\mu = 1\text{GeV}$ )

$$\lambda_{cr}(1\text{GeV}) = -0.91 \pm 0.06 \quad . \quad (2.21)$$

A linear combination of  $a_1$  and  $a_2$  is fixed by the measured  $K^+ \rightarrow \pi^+ e^+ e^-$  decay amplitude. Fortunately it is the same combination of  $a_1$  and  $a_2$  that enters into the  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  decay amplitude. The one-photon part of the  $K^+ \rightarrow \pi^+ e^+ e^-$  decay amplitude can be written in terms of a single form factor  $f(q^2)$

$$M^{(1\gamma)}(K^+ \rightarrow \pi^+ e^+ e^-) = \frac{s_1 G_F}{\sqrt{2}} \frac{\alpha}{4\pi} f(q^2) p_\pi^\mu \bar{u}(k_-) \gamma_\mu v(k_+) \quad . \quad (2.22)$$

The one-loop diagrams in fig. 3 and the operators in (2.17) and (2.18) give [10]

$$\begin{aligned} f(q^2) &= 2g_8 \left( \phi_K(q^2) + \phi_\pi(q^2) - \frac{1}{6} \ln(m_K^2/\mu^2) - \frac{1}{6} \ln(m_\pi^2/\mu^2) \right. \\ &\quad \left. + \frac{2}{3} (a_1(\mu) + 2a_2(\mu)) - 4\lambda_{cr}(\mu) + \frac{1}{3} \right) \quad , \quad (2.23) \\ &= 2g_8 ( \phi_K(q^2) + \phi_\pi(q^2) + w_+ ) \end{aligned}$$

where

$$\phi_i(q^2) = \int_0^1 dx \left( \frac{m_i^2}{q^2} - x(1-x) \right) \ln \left( 1 - \frac{q^2}{m_i^2} x(1-x) \right) \quad . \quad (2.24)$$

This relation defines the  $\mu$  independent constant  $w_+$  [10] which has been experimentally determined to be [11]

$$w_+ = 0.89_{-0.14}^{+0.24} \quad . \quad (2.25)$$

Using the central values of  $\lambda_{cr}(1\text{GeV})$  and  $w_+$  we find that

$$a_1(1\text{GeV}) + 2a_2(1\text{GeV}) = -6.0 \quad , \quad (2.26)$$

with an associated error around 10% (which is correlated with the uncertainty in  $\lambda_{cr}(1\text{GeV})$ ). Throughout the remainder of this work we will use the central values of  $\lambda_{cr}(1\text{GeV})$  and  $a_1(1\text{GeV}) + 2a_2(1\text{GeV})$  and suppress the associated uncertainties. Note



that the contributions  $\lambda_{cr}(1\text{GeV})$  and  $(a_1 + 2a_2)(1\text{GeV})$  to  $f(q^2)$  are separately quite large but they almost cancel against each other.

At  $\mathcal{O}(p^4)$  the form factors  $F_{\pm}^{(2)}$  for  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  decay follow from the Feynman diagrams in fig. 4 and tree level matrix elements of the operators in (2.17) and (2.18). We find, using  $\overline{MS}$  subtraction, that

$$F_-^{(2)} = g_8 \left( -\frac{2}{3}q^2[a_1(\mu) + 2a_2(\mu) + 6a_3(\mu) - 6a_4(\mu)] - 4q^2\lambda_{cr}(\mu) + \frac{2}{3}q^2 + \phi_{K\eta} + \phi_{K\pi} \right. \\ \left. - 4 \int_0^1 dx \left[ q^2 x(1-x) \ell n \left( \frac{m_\pi^2 - q^2 x(1-x)}{\mu^2} \right) - m_\pi^2 \ell n \left( 1 - \frac{q^2 x(1-x)}{m_\pi^2} \right) \right] \right. \\ \left. + 2q^2 \left( \frac{m_K^2 - m_\pi^2}{q^2 + 2q \cdot (p_+ + p_-)} \right) \left( \phi_K(q^2) - \phi_\pi(q^2) + \frac{1}{6} \ell n \left( \frac{m_\pi^2}{m_K^2} \right) \right) \right) \quad (2.27)$$

where

$$\phi_{K\eta} = \frac{2}{9}(m_K^2 - m_\pi^2)^2 \left( \int_0^1 dy y \int_0^{1-y} dx \frac{1}{\mu_1^2} \right. \\ \left. + \frac{1}{(q^2 + 2q \cdot p_-)} \int_0^1 dx \ell n \left( 1 - \frac{x(1-x)(q^2 + 2q \cdot p_-)}{m_K^2(1-x) + m_\eta^2 x - m_\pi^2 x(1-x)} \right) \right) \\ + \frac{1}{3}(m_K^2 - m_\pi^2) \left( 2 \int_0^1 dy \int_0^{1-y} dx \ell n \left( 1 - \frac{(q^2 x(1-x) + 2q \cdot p_- xy)}{m_K^2(1-y) + m_\eta^2 y - m_\pi^2 y(1-y)} \right) \right. \\ \left. + 3 \int_0^1 dy \int_0^{1-y} dx \ell n \left( 1 - \frac{(q^2 x(1-x) + 2q \cdot p_+ xy)}{m_K^2(1-y) + m_\eta^2 y - m_\pi^2 y(1-y)} \right) \right. \\ \left. + \int_0^1 dy y \int_0^{1-y} \frac{1}{\mu_1^2} \left[ (q(1-x) + p_- y) \cdot (4q + 6p_+ + 4p_-) - 2m_K^2 - 2p_+ \cdot (q + p_-) \right] \right. \\ \left. + \int_0^1 dx \left( 1 + x + \frac{(x-1)(3m_K^2 - 2m_\pi^2)}{q^2 + 2p_- \cdot q} \right) \cdot \ell n \left( 1 - \frac{x(1-x)(q^2 + 2q \cdot p_-)}{m_K^2(1-x) + m_\eta^2 x - m_\pi^2 x(1-x)} \right) \right) \quad (2.28)$$

with

$$\mu_1^2 = m_K^2(1-y) + m_\eta^2 y - m_\pi^2 y(1-y) - q^2 x(1-x) - 2q \cdot p_- xy \quad (2.29)$$

The other function is

$$\begin{aligned}
\phi_{K\pi} = (m_K^2 - m_\pi^2) & \left( \int_0^1 dy \int_0^{1-y} dx \ell n \left( 1 - \frac{(q^2 x(1-x) + 2q \cdot p_- xy)}{m_K^2 y + m_\pi^2 (1-y)^2} \right) \right. \\
& + \int_0^1 dy \int_0^{1-y} dx \ell n \left( 1 - \frac{q^2 x(1-x) + 2q \cdot p_+ xy}{m_K^2 y + m_\pi^2 (1-y)^2} \right) \\
& + 2 \int_0^1 dy y \int_0^{1-y} dx \frac{(p_+ + p_- + q) \cdot (q(1-x) + yp_- - p_+)}{\mu_2^2} \\
& \left. + \int_0^1 dx \left( 1 + x + \frac{m_K^2(x-1)}{q^2 + 2q \cdot p_-} \right) \ell n \left( 1 - \frac{x(1-x)(q^2 + 2q \cdot p_-)}{m_\pi^2 (1-x)^2 + m_K^2 x} \right) \right) \quad (2.30)
\end{aligned}$$

with

$$\mu_2^2 = m_K^2 y + m_\pi^2 (1-y)^2 - q^2 x(1-x) - 2q \cdot p_- xy \quad . \quad (2.31)$$

The Gell-Mann–Okubo mass formula (2.6) has been used to simplify some of the dependence on the pseudoscalar masses in (2.28) and (2.30).  $F_+^{(2)}$  is obtained from (2.27) by taking  $p_+ \rightarrow p_-$  and  $p_- \rightarrow p_+$ . Notice that the combination of coefficients  $(a_1 + 2a_2)$  and  $\lambda_{cr}$  that appear in the expression for  $F_\pm^{(2)}$  has a relative sign difference compared to the combination that appears in the expression for  $f(s)$  given in eq.(2.23). The uncertainty in  $\lambda_{cr}(1\text{GeV})$  and  $w_+$  gives rise to about a 10% uncertainty in the combination of counterterms that appears in (2.27). The one photon part of the  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  decay amplitude is the largest and dominates the rate. In the next section we use the form factors calculated here to obtain  $d\Gamma(K_L \rightarrow \pi^+ \pi^- e^+ e^-)/dq^2$ . One (scale independent) linear combination of counterterms  $w_L = a_3 - a_4$  is not determined by the present experimental data and consequently we cannot predict the rate for  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ . However, this is the only undetermined constant and the entire function  $d\Gamma(K_L \rightarrow \pi^+ \pi^- e^+ e^-)/dq^2$  is experimentally accessible.

### 3. The Differential Decay Rate

The  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  decay rate is obtained by squaring the invariant matrix element (1.1), summing over the  $e^+$  and  $e^-$  spins, and integrating over the phase space. Since the  $e^+$  and  $e^-$  four momenta only occur in the lepton trace,  $\text{Tr} [\not{k}_- \gamma_\nu \not{k}_+ \gamma_\mu]$ , the phase space integrations over  $k_-$  and  $k_+$  produce a factor

$$\begin{aligned}
& \int \frac{d^3 k_-}{(2\pi)^3 2k_-^0} \int \frac{d^3 k_+}{(2\pi)^3 2k_+^0} (2\pi)^4 \delta^4(q - k_- - k_+) \text{Tr} [\not{k}_- \gamma_\nu \not{k}_+ \gamma_\mu] \\
& = \frac{1}{6\pi} (q_\mu q_\nu - q^2 g_{\mu\nu}) \quad . \quad (3.1)
\end{aligned}$$

The remaining phase space integrations can be taken to be over  $q^2$  and the sum and difference of the pion energies in the  $K_L$  rest frame,  $E_S = p_+^0 + p_-^0, E_D = p_+^0 - p_-^0$ . The contribution of the form factors  $F_\pm$  and  $G$  to  $d\Gamma/dq^2$  do not interfere. Therefore, we can write

$$\frac{d\Gamma}{dq^2}(K_L \rightarrow \pi^+\pi^-e^+e^-) = \frac{d\Gamma_G}{dq^2} + \frac{d\Gamma_F}{dq^2} \quad , \quad (3.2)$$

where

$$\begin{aligned} \frac{d\Gamma_G}{dq^2} &= \frac{G_F^2 \alpha^2 s_1^2}{m_K f^2 2^6 (2\pi)^7 3 q^2} \int dE_S \int dE_D |G|^2 \\ &\quad \left[ m_\pi^4 q^2 - m_\pi^2 (p_- \cdot q)^2 - m_\pi^2 (p_+ \cdot q)^2 + 2(p_+ \cdot p_-)(q \cdot p_+)(q \cdot p_-) - q^2 (p_+ \cdot p_-)^2 \right] \\ \frac{d\Gamma_F}{dq^2} &= \frac{G_F^2 \alpha^2 s_1^2}{m_K f^2 2^6 (2\pi)^7 3 q^4} \int dE_S \int dE_D \left[ |F_+ q \cdot p_+ + F_- q \cdot p_-|^2 \right. \\ &\quad \left. - q^2 (|F_+|^2 m_\pi^2 + |F_-|^2 m_\pi^2 + 2\text{Re}(F_+ F_-^*) p_+ \cdot p_-) \right] \end{aligned} \quad (3.3)$$

In eq. (3.2) and eq. (3.3) the difference of pion energies is integrated over the region  $-E_D^{(max)} < E_D < E_D^{(max)}$  where

$$E_D^{(max)} = \sqrt{\frac{2m_K E_S + q^2 - m_K^2 - 4m_\pi^2}{2m_K E_S + q^2 - m_K^2}} \sqrt{(m_K - E_S)^2 - q^2} \quad , \quad (3.4)$$

and the sum of pion energies is integrated over the region  $E_S^{(min)} < E_S < E_S^{(max)}$  where the boundaries are

$$\begin{aligned} E_S^{(max)} &= m_K - \sqrt{q^2} \\ E_S^{(min)} &= \frac{m_K^2 - q^2 + 4m_\pi^2}{2m_K} \end{aligned} \quad . \quad (3.5)$$

The scalar products appearing in the expression for the rates are easily expressed in terms of  $E_S, E_D$  and  $q^2$ :

$$\begin{aligned} p_+ \cdot p_- &= \frac{1}{2}(q^2 - m_K^2 - 2m_\pi^2 + 2m_K E_S) \\ q \cdot p_+ &= \frac{1}{2}(-m_K E_S + m_K E_D - q^2 + m_K^2) \\ q \cdot p_- &= \frac{1}{2}(-m_K E_S - m_K E_D - q^2 + m_K^2) \end{aligned} \quad . \quad (3.6)$$

The form factors  $F_{\pm}^{(1)}$  and  $F_{\pm}^{(2)}$  have the opposite property under interchange of pion momenta and consequently they do not interfere in  $d\Gamma/dq^2$ . Neglecting terms in chiral expansion of  $\mathcal{O}(p^6)$  and higher the differential decay rate given in (3.2) becomes

$$\frac{d\Gamma}{dq^2}(K_L \rightarrow \pi^+\pi^-e^+e^-) = \frac{d\Gamma_{G^{(2)}}}{dq^2} + \frac{d\Gamma_{F^{(1)}}}{dq^2} + \frac{d\Gamma_{F^{(2)}}}{dq^2} \quad . \quad (3.7)$$

In fig. 5 we have graphed for each of the three terms on the right hand side of (3.7)

$$\frac{1}{\Gamma_{K_L}} \frac{d\Gamma}{dy} = 2y (m_K - 2m_\pi)^2 \frac{1}{\Gamma_{K_L}} \frac{d\Gamma}{dq^2} \quad , \quad (3.8)$$

where  $y = \sqrt{q^2}/(m_K - 2m_\pi)$ ,  $\Gamma_{K_L}$  is the total width of the  $K_L$  and we have set  $w_L = 0$ .

Integrating the three terms on the rhs of (3.7) over the invariant mass interval  $q^2 > (30MeV)^2$  (corresponding to  $y > 0.14$ ) we find that for  $w_L = 0$

$$10^8 \cdot Br(K_L \rightarrow \pi^+\pi^-e^+e^-; q^2 > (30MeV)^2) = 3.8 + 0.78 + 3.4 = 8.0 \quad . \quad (3.9)$$

The branching fraction over this range of  $e^+e^-$  invariant mass is dominated by the region of low  $q^2$  and for typical values of  $w_L$  it receives comparable contributions from the form factors  $G$  and  $F^{(2)}$ . However, in the region of high  $q^2$  the branching fraction is likely to be dominated by the  $F_{\pm}^{(2)}$  form factor. For  $q^2 > (80MeV)^2$  (corresponding to  $y > 0.37$ ) and  $w_L = 0$  the three terms on the rhs of (3.7) contribute

$$10^8 \cdot Br(K_L \rightarrow \pi^+\pi^-e^+e^-; q^2 > (80MeV)^2) = 0.61 + 0.07 + 1.9 = 2.6 \quad . \quad (3.10)$$

A summary of our results for the rate can be found in Table 1. We have displayed the contribution to the branching ratio (in units of  $10^{-8}$ ) from the three form factors  $G$ ,  $F^{(1)}$  and  $F^{(2)}$  for different values of the minimum lepton pair invariant mass  $q_{\min}^2$ . Since the loop contribution to the form factor  $F_{\pm}^{(2)}$  is small, it will be difficult to extract a unique value for  $w_L$  from  $d\Gamma/dq^2$  data alone; a two-fold ambiguity in the value of  $w_L$  will persist.

Lower cut $q_{\min}^2$	$Br(10^{-8})_G$	$Br(10^{-8})_{F^{(1)}}$	$Br(10^{-8})_{F^{(2)}}$
$(10\text{MeV})^2$	8.8	3.3	$3.6 - 3.4w_L + 0.8w_L^2$
$(20\text{MeV})^2$	5.6	1.5	$3.5 - 3.3w_L + 0.8w_L^2$
$(30\text{MeV})^2$	3.8	0.8	$3.4 - 3.2w_L + 0.8w_L^2$
$(40\text{MeV})^2$	2.7	0.5	$3.1 - 3.0w_L + 0.7w_L^2$
$(60\text{MeV})^2$	1.3	0.2	$2.6 - 2.4w_L + 0.6w_L^2$
$(80\text{MeV})^2$	0.6	0.07	$1.9 - 1.8w_L + 0.4w_L^2$
$(100\text{MeV})^2$	0.3	0.03	$1.3 - 1.2w_L + 0.3w_L^2$
$(120\text{MeV})^2$	0.1	0.01	$0.74 - 0.68w_L + 0.16w_L^2$
$(180\text{MeV})^2$	0.00072	0.0001	$0.027 - 0.025w_L + 0.006w_L^2$

**Table 1: Contributions to the Branching Ratio ( $10^{-8}$ ) for a range of  $q_{\min}^2$**

#### 4. The $Z$ -penguin and $W$ -box Amplitude

The short distance  $W$ -box and  $Z$ -penguin diagrams give the effective Lagrange density

$$\mathcal{L}_{SD} = \xi \frac{s_1 G_F \alpha}{\sqrt{2}} \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{e} \gamma^\mu \gamma_5 e + h.c. \quad . \quad (4.1)$$

Here we only keep the part that contains the lepton axial current (the vector current is neglected). It is only the axial current that gives rise to observables that are antisymmetric under interchange of  $e^+$  and  $e^-$  four momenta,  $k_+ \leftrightarrow k_-$ .

In (4.1) the quantity  $\xi$  receives significant contributions from both the top quark and charm quark loops and is given by

$$\xi = -\tilde{\xi}_c + \left( \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \right) \tilde{\xi}_t \quad , \quad (4.2)$$

where

$$\tilde{\xi}_q = \tilde{\xi}_q^{(Z)} + \tilde{\xi}_q^{(W)} \quad , \quad (4.3)$$

is the sum of the contributions of the  $Z$ -penguin and  $W$ -box diagrams. It is convenient to express the combination of elements of the Cabibbo–Kobayashi–Maskawa matrix that enters in  $\xi$  in terms of  $|V_{cb}|$  and the standard coordinates  $\rho + i\eta$  of the unitarity triangle

$$V_{ts}^* V_{td} / V_{us}^* V_{ud} = (\rho - 1 + i\eta) |V_{cb}|^2 \quad . \quad (4.4)$$

A value of  $|V_{cb}| \simeq 0.04$  is obtained from inclusive  $B \rightarrow X_c e \bar{\nu}_e$  decay and from exclusive  $B \rightarrow D^* e \bar{\nu}_e$  decay. Although the values of  $\rho$  and  $\eta$  are not determined by present data, they are expected to be of order unity.

The quantities  $\tilde{\xi}_c$  and  $\tilde{\xi}_t$  have been calculated including perturbative QCD corrections at the next to leading logarithmic level [12] [13]. There is some sensitivity to the values of  $\Lambda_{QCD}$ ,  $m_c$  and  $m_t$  but  $\tilde{\xi}_c$  is of order  $10^{-4}$  and  $\tilde{\xi}_t$  is of order unity.

The quark-level Lagrange density in eq. (4.3) can be converted into a Lagrange density involving the  $\pi, K$  and  $\eta$  hadrons using the Noether procedure. Equating the QCD chiral currents with those obtained from chiral variations of the effective lagrangian in eq. (2.4) leads to

$$\mathcal{L}_{SD} = -\xi \frac{iG_F \alpha s_1}{2\sqrt{2}} f^2 \text{Tr}(\partial^\mu \Sigma \Sigma^\dagger T) \bar{e} \gamma_\mu \gamma_5 e + h.c. \quad (4.5)$$

Expanding out  $\Sigma$  in terms of the meson fields  $M$  we find that the Lagrange density (4.5) implies that the short distance contribution to the  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  decay amplitude from the  $W$ -box and  $Z$ -penguin diagrams is

$$M^{(SD)} = \frac{G_F s_1 \alpha}{f} (\xi p_-^\mu + \xi^* p_+^\mu) \bar{u}(k_-) \gamma_\mu \gamma_5 v(k_+) \quad (4.6)$$

## 5. The Asymmetry $A_{CP}$

It is the interference of  $M^{(SD)}$  in (4.6) with  $M^{(1\gamma)}$  in (1.1) that produces the asymmetry  $A_{CP}$  defined in (1.8). For calculation of  $A_{CP}$  it is convenient to use the phase space variables used by Pais and Treiman [14] for  $K_{\ell 4}$  decay (rather than those used for the total rate in Section 3). They are:  $q^2 = (k_+ + k_-)^2$ ;  $s = (p_+ + p_-)^2$ ;  $\theta_\pi$  the angle formed by the  $\pi^+$  three momentum and the  $K_L$  three-momentum in the  $\pi^+ \pi^-$  rest frame;  $\theta_\ell$ , the angle between the  $e^-$  three momentum and the  $K_L$  three momentum in the  $e^+ e^-$  rest frame;  $\phi$ , the angle between the normals to the planes defined in the  $K_L$  rest frame by the  $\pi^+ \pi^-$  pair and the  $e^+ e^-$  pair. In terms of these variables

$$\frac{(\vec{p}_- \times \vec{p}_+) \cdot (\vec{k}_- - \vec{k}_+)}{|\vec{p}_- \times \vec{p}_+| \cdot |\vec{k}_- - \vec{k}_+|} = \text{sign}(\sin \phi) \quad , \quad (5.1)$$

and the asymmetry is

$$A_{CP} = \frac{1}{2^7 (2\pi)^6 m_K^3 \Gamma_{K_L}} \left( \int_0^{2\pi} d\phi \text{sign}(\sin \phi) \right) \int dc_\pi dc_e ds dq^2 \beta X \text{Re} \left( M^{(SD)*} M^{(1\gamma)} \right) \quad , \quad (5.2)$$

where  $c_\pi = \cos \theta_\pi$ ,  $c_e = \cos \theta_e$ . The other kinematic functions appearing in this expression are

$$\begin{aligned}\beta &= [1 - 4m_\pi^2/s]^{1/2} \\ X &= \left[ \left( \frac{m_K^2 - s - q^2}{2} \right)^2 - sq^2 \right]^{1/2} .\end{aligned}\tag{5.3}$$

In order to evaluate the contributing form factors the following scalar products of four vectors are required:

$$\begin{aligned}q \cdot p_+ &= \frac{1}{4}(m_K^2 - s - q^2) - \frac{1}{2}\beta X \cos \theta_\pi \\ q \cdot p_- &= \frac{1}{4}(m_K^2 - s - q^2) + \frac{1}{2}\beta X \cos \theta_\pi \\ p_+ \cdot p_- &= \frac{1}{2}(s - 2m_\pi^2) \\ \varepsilon_{\alpha\beta\sigma\rho} p_+^\alpha p_-^\beta k_+^\sigma k_-^\rho &= -\frac{1}{4}\beta X \sqrt{sq^2} \sin \theta_e \sin \theta_\pi \sin \phi\end{aligned}\tag{5.4}$$

If the variables  $s$  and  $q^2$  are not integrated over the complete phase space then it is understood that the same is to be done for the  $K_L$  width  $\Gamma_{K_L}$  in the denominator of eq. (5.2).

The form factor  $G$  does not enter into  $Re(M^{(SD)*} M^{(1\gamma)})$  (a sum over  $e^+$  and  $e^-$  spins is understood). Integrating our  $\cos \theta_e$  and  $\phi$  we find that

$$\begin{aligned}A_{CP} &= \frac{G_F^2 s_1^2 \alpha^2}{2^8 (2\pi)^6 f^2 m_K^3 \Gamma_{K_L}} \int dc_\pi ds dq^2 \sin \theta_\pi \\ &\quad \beta^2 X^2 \sqrt{\frac{s}{q^2}} \left[ Im(\xi) (Re(F_+) + Re(F_-)) + Re(\xi) (Im(F_+) - Im(F_-)) \right] .\end{aligned}\tag{5.5}$$

The integration over  $\cos \theta_\pi$  implies that at leading non-trivial order of chiral perturbation theory  $Im(F_+) - Im(F_-) \rightarrow Im(F_+^{(1)}) - Im(F_-^{(1)})$  reflecting indirect CP violation from  $\epsilon$  and  $Re(F_+) + Re(F_-) \rightarrow Re(F_+^{(2)}) + Re(F_-^{(2)})$  in eq.(5.5) .

Using (4.2) and (4.4) we can write the CP violating asymmetry in terms of the real and imaginary parts of the CKM elements

$$A_{CP} = A_1 \left( (\rho - 1) |V_{cb}|^2 \tilde{\xi}_t - \tilde{\xi}_c \right) - A_2 \eta |V_{cb}|^2 \tilde{\xi}_t \quad ,\tag{5.6}$$

where  $A_1$  arises from indirect CP nonconservation (i.e.  $\bar{K}^0 - K^0$  mixing ) and  $A_2$  arises from direct CP nonconservation. We are only able to predict  $|A_{CP}|$  since the sign of  $g_8$  is not known. Our expressions for  $F_\pm^{(1)}$  and  $F_\pm^{(2)}$  with  $w_L = 0$  give (up to an overall sign)

$$A_1 = 2.7 \times 10^{-2} \quad , \quad A_2 = 3.9 \times 10^{-2} \quad ,\tag{5.7}$$

for  $q^2 \geq (30\text{MeV})^2$  and

$$A_1 = 2.4 \times 10^{-2} \quad , \quad A_2 = 8.4 \times 10^{-2} \quad , \quad (5.8)$$

for  $q^2 \geq (80\text{MeV})^2$ . In Table 2 we present  $A_1$  and  $A_2$  for a range of values of the minimum lepton pair invariant mass,  $q_{\min}^2$ , normalized to the branching ratios given in Table 1 assuming  $w_L = 0$ .

Lower cut $q_{\min}^2$	$A_1$	$A_2$
$(10\text{MeV})^2$	$2.0 \times 10^{-2}$	$2.0 \times 10^{-2}$
$(20\text{MeV})^2$	$2.5 \times 10^{-2}$	$3.0 \times 10^{-2}$
$(30\text{MeV})^2$	$2.7 \times 10^{-2}$	$3.9 \times 10^{-2}$
$(40\text{MeV})^2$	$2.8 \times 10^{-2}$	$4.8 \times 10^{-2}$
$(60\text{MeV})^2$	$2.7 \times 10^{-2}$	$6.8 \times 10^{-2}$
$(80\text{MeV})^2$	$2.4 \times 10^{-2}$	$8.4 \times 10^{-2}$
$(100\text{MeV})^2$	$2.1 \times 10^{-2}$	$9.8 \times 10^{-2}$
$(120\text{MeV})^2$	$1.8 \times 10^{-2}$	0.11
$(180\text{MeV})^2$	$1.3 \times 10^{-2}$	0.13

**Table 2: The CP violating quantities  $A_1$ ,  $A_2$  with  $w_L = 0$  for different values of  $q_{\min}^2$**

We find that direct and indirect sources of CP nonconservation give comparable contributions to  $A_{CP}$ . In our computation we have neglected final state  $\pi\pi$  interactions which are formally higher order in chiral perturbation theory. With the values of  $A_1$  and  $A_2$  given in Table 2,  $|A_{CP}|$  is only of order  $10^{-4}$  and further refinements of our calculation do not seem warranted.

## 6. The Asymmetry $B_{CP}$

Using the kinematic variables introduced in the previous section the CP violating observable  $B_{CP}$  is defined as

$$B_{CP} = \langle \text{sign}(\sin \phi \cos \phi) \rangle \quad . \quad (6.1)$$



At leading order in chiral perturbation theory it arises from the interference of  $F_{\pm}^{(1)}$  with  $G^{(2)}$ . The CP violating form factors  $F_{\pm}^{(1)}$  are not small because they occur at a lower order in chiral perturbation theory than the other form factors,  $F_{\pm}^{(2)}$  and  $G^{(2)}$ . Consequently, as was noted in refs. [2] and [3],  $B_{CP}$  is quite large. Neglecting  $M^{(SD)}$  we find after integrating over  $\phi$  and  $\cos\theta_e$  that

$$B_{CP} = \frac{G_F^2 s_1^2 \alpha^2}{3 \cdot 2^7 (2\pi)^8 f^2 m_K^3 \Gamma_{K_L}} \int dc_{\pi} ds dq^2 \sin^2 \theta_{\pi} \beta^3 X^2 \frac{s}{q^2} \text{Im} [G (F_+^* - F_-^*)] \quad . \quad (6.2)$$

If the variables  $s$  and  $q^2$  are not integrated over the entire phase space then it is understood that the same is to be done to the  $K_L$  width  $\Gamma_{K_L}$  in the denominator of (6.2). The form factor  $G$  is real at leading order in chiral perturbation theory and the imaginary part arises from the phase in  $F_+ - F_-$  induced by  $K^0 - \bar{K}^0$  mixing. The integration over  $\cos\theta_{\pi}$  implies that  $F_+ - F_- \rightarrow F_+^{(1)} - F_-^{(1)}$  in eq.(6.2). Using our expressions for  $F_{\pm}^{(1)}$  and the value of  $|G^{(2)}|$  we find that with  $w_L = 0$   $|B_{CP}| \simeq 6.3\%$  for  $q^2 > (30 \text{ MeV})^2$  and  $|B_{CP}| \simeq 2.4\%$  for  $q^2 > (80 \text{ MeV})^2$ . The asymmetry for a range of values of  $q_{\min}^2$  are shown in Table 3.

Lower cut $q_{\min}^2$	$ B_{CP} \cdot Br(10^{-8}) $ (%)
$(10\text{MeV})^2$	134
$(20\text{MeV})^2$	78
$(30\text{MeV})^2$	50
$(40\text{MeV})^2$	33
$(60\text{MeV})^2$	14
$(80\text{MeV})^2$	6.3
$(100\text{MeV})^2$	2.5
$(120\text{MeV})^2$	0.92
$(180\text{MeV})^2$	0.0086

**Table 3: The CP violating observable  $|B_{CP} \cdot Br(10^{-8})|$  for a range of values of  $q_{\min}^2$**

Note that in Table 3  $Br(10^{-8})$  denotes the  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  branching ratio in units of  $10^{-8}$  with the same cut on  $q^2$  imposed. We have neglected final state  $\pi\pi$  interactions because they arise at higher order in chiral perturbation theory. Our prediction for  $|B_{CP}|$  has considerable uncertainty because of the neglect of final state  $\pi\pi$  interactions and because neglected  $\mathcal{O}(p^6)$  contributions to  $G$  seem to be important.

## 7. Conclusions

In this paper we have calculated the one-photon contribution to the  $K_L \rightarrow \pi^+\pi^-e^+e^-$  decay rate. We used chiral perturbation theory to determine the form factors and for  $e^+e^-$  pairs with high invariant mass ( $q^2 \gg 4m_e^2$ ) found important new contributions that were not included in previous work [2][3]. The amplitude for  $K_L \rightarrow \pi^+\pi^-e^+e^-$  depends on the undetermined (renormalization scale independent) combination of counterterms  $w_L$ . We found that for  $q^2 = (k_+ + k_-)^2 > (30 \text{ MeV})^2$  the branching ratio for  $K_L \rightarrow \pi^+\pi^-e^+e^-$  is approximately  $(8.0 - 3.2w_L + 0.8w_L^2) \times 10^{-8}$  and for  $q^2 > (80 \text{ MeV})^2$  the branching ratio is approximately  $(2.6 - 1.8w_L + 0.4w_L^2) \times 10^{-8}$ .

One interesting aspect of this decay mode is that the CP even component of the  $K_L$  state contributes at a lower order in chiral perturbation theory than the CP odd component. This enhances CP violating effects in  $K_L \rightarrow \pi^+\pi^-e^+e^-$  decay. For example, the CP violating observable [2] [3]  $B_{CP} = \langle \text{sign}(\sin \phi \cos \phi) \rangle$ , where  $\phi$  is the angle between the normals to the  $\pi^+\pi^-$  and  $e^+e^-$  planes, is about 6% for  $q^2 > (30 \text{ MeV})^2$  if  $w_L = 0$ . The CP violating observable  $A_{CP} = \langle \text{sign}(\sin \phi) \rangle$  arises from the interference of W-box and Z-penguin amplitudes with the one-photon part of the decay amplitude. Unfortunately, we find that  $A_{CP}$  is of order  $10^{-4}$  and hence most likely unmeasurable.

Chiral Perturbation theory has been extensively applied to nonleptonic, semileptonic and radiative kaon decays. The study of  $K_L \rightarrow \pi^+\pi^-e^+e^-$  offers an opportunity to determine the linear combination of coefficients in the  $\mathcal{O}(p^4)$  chiral lagrangian that we call  $w_L$  and to test the applicability of  $\mathcal{O}(p^4)$  chiral perturbation theory for kaon decay.

Some improvements in our calculations are possible. While a full computation of the  $\mathcal{O}(p^6)$  contribution to  $F_\pm$  and  $G$  arising from two-loop diagrams and new local operators does not seem feasible it should be possible to calculate the leading contribution to the absorptive parts of  $G$  and  $F_+ - F_-$ . Note that the absorptive parts come from both  $\pi\pi \rightarrow \pi\pi$  rescattering and because of CP nonconservation from  $\pi\pi \rightarrow \pi\pi\gamma$ . We hope to present results for this in a future publication.

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## Figure Captions

- Fig. 1. Feynman diagrams contributing to  $F_{\pm}^{(1)}$ .
- Fig. 2. Feynman diagrams contributing to the  $\pi^{\pm}$  charge radius,  $\langle r_{\pi}^2 \rangle$ , at leading order in chiral perturbation theory.
- Fig. 3. The Feynman diagrams contributing to the amplitude for  $K^+ \rightarrow \pi^+ \gamma^*$  at leading order in chiral perturbation theory. The solid square denotes a vertex from the gauged weak lagrangian in (2.7), the solid circle denotes a vertex from the gauged strong lagrangian in (2.4). Figures 3(a) involve only weak and electromagnetic vertices while Figures 3(b) also has a strong vertex. Figures 3(c) are the contributions from the kaon and pion charge radii (including both loop graphs and the tree-level counterterm). Figure 3(d) is the contribution of the weak counterterm as given by (2.18). We have not shown the wavefunction renormalization of the tree graphs for the process as the sum of these graphs vanish.
- Fig. 4. Feynman diagrams contributing to the CP conserving amplitude for  $K_L \rightarrow \pi^+ \pi^- \gamma^*$  at leading order in chiral perturbation theory. The notation is the same as in fig. 3 and we have not shown the wavefunction renormalization of the tree graphs for the process as the sum of these graphs vanish.
- Fig. 5. The differential decay spectrum as a function of  $y$  the invariant mass of the lepton pair normalized to  $m_K - 2m_{\pi}$ . The dot-dashed curve is the contribution from  $F_{\pm}^{(1)}$ , the dotted curve is the contribution from  $F_{\pm}^{(2)}$  with  $w_L = 0$  and the dashed curve is the contribution from  $G^{(2)}$ . The total differential decay rate for  $w_L = 0$  is given by the solid curve.

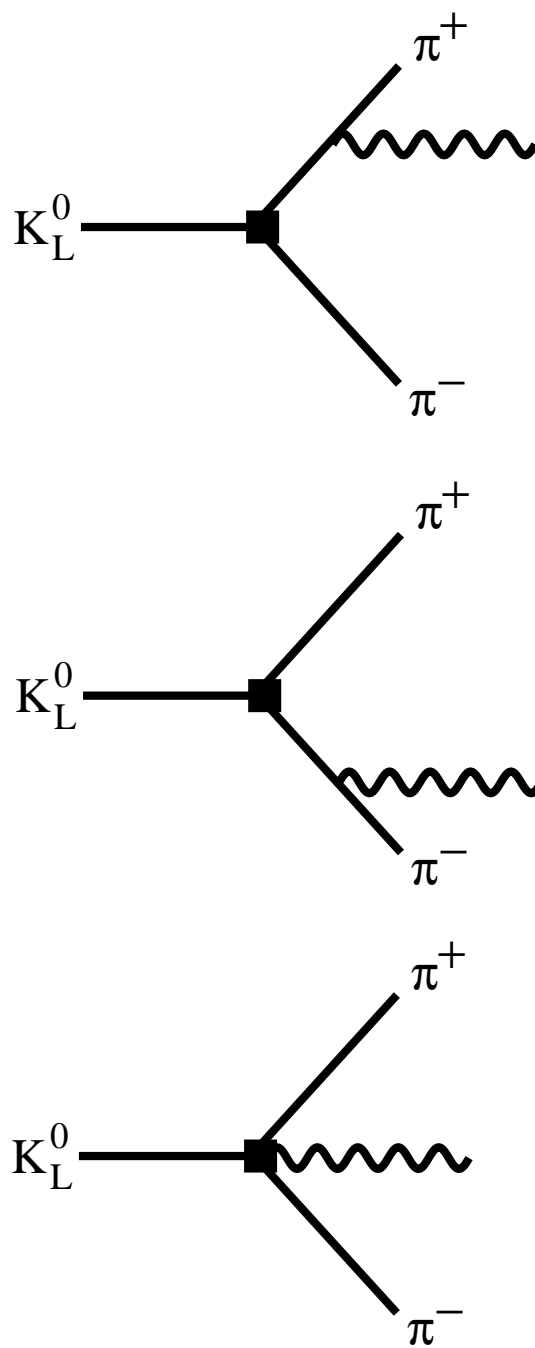


Figure 1

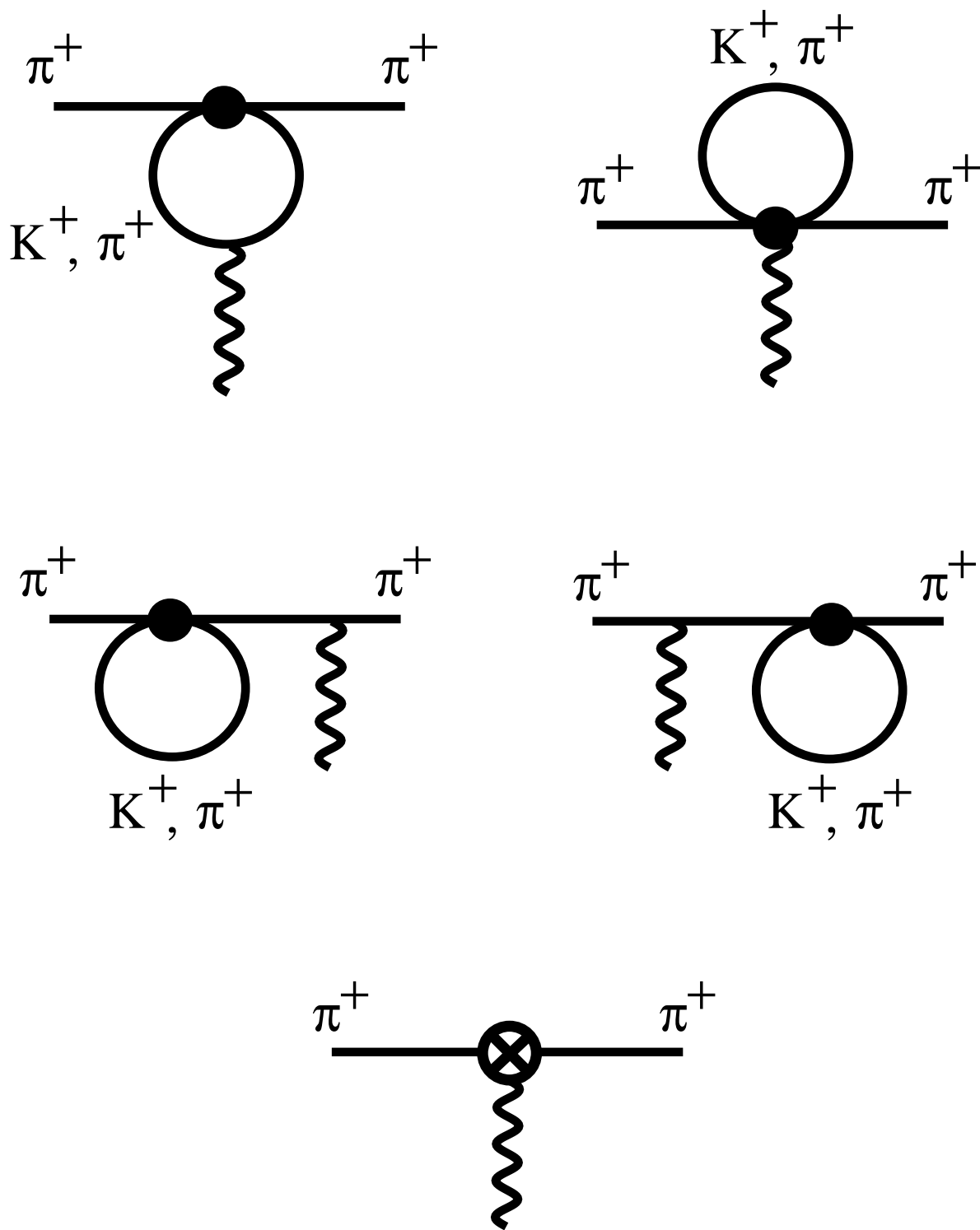


Figure 2

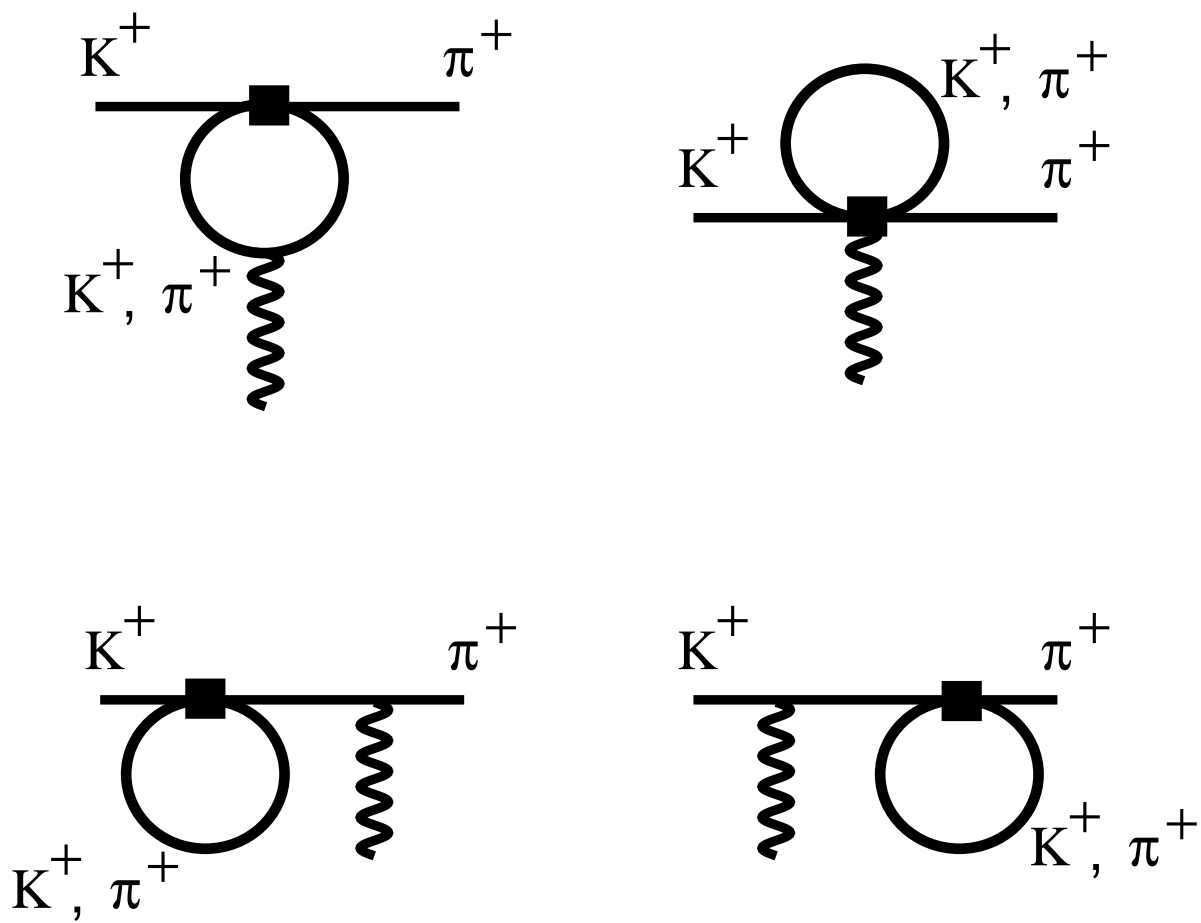


Figure 3(a)

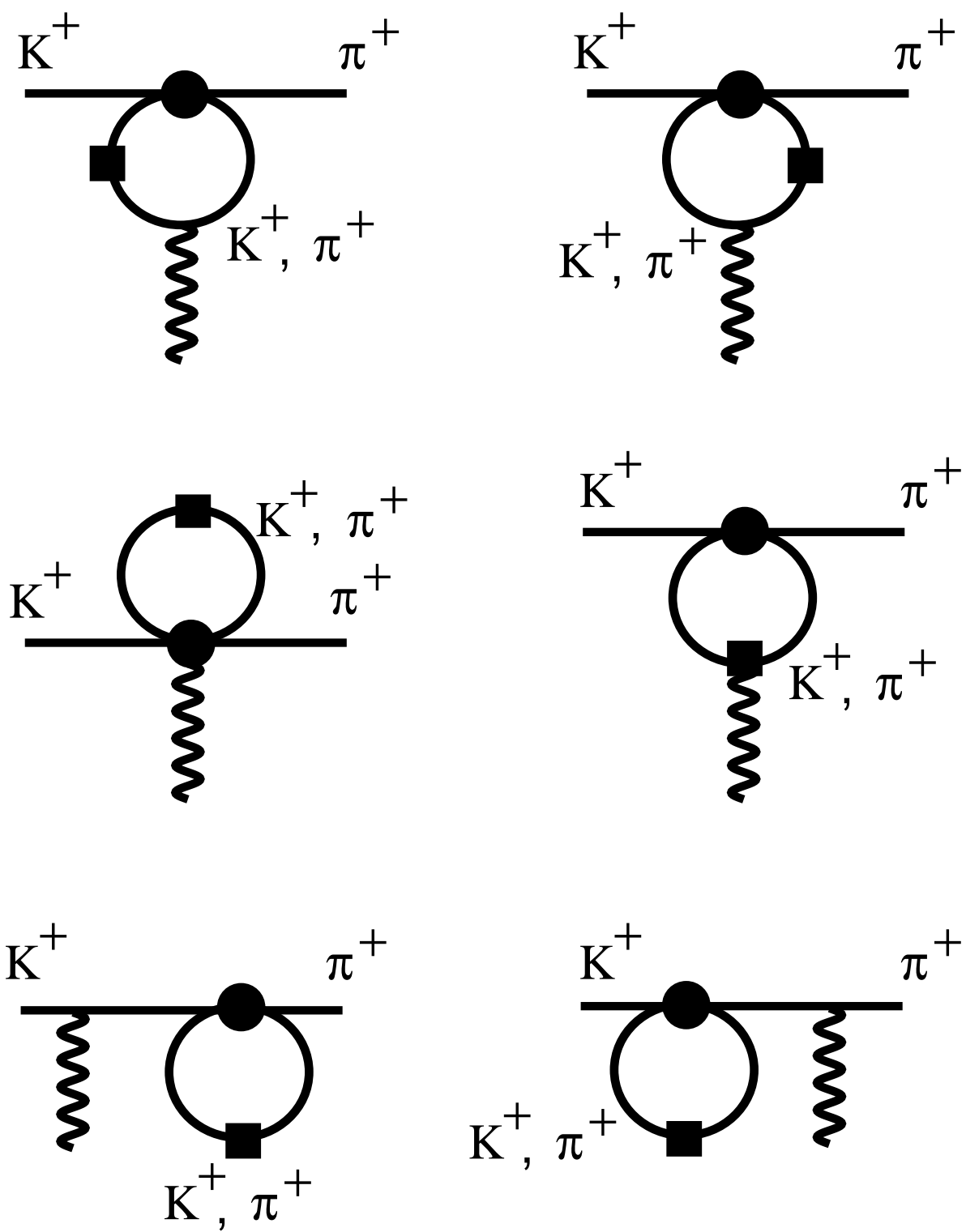


Figure 3(b)



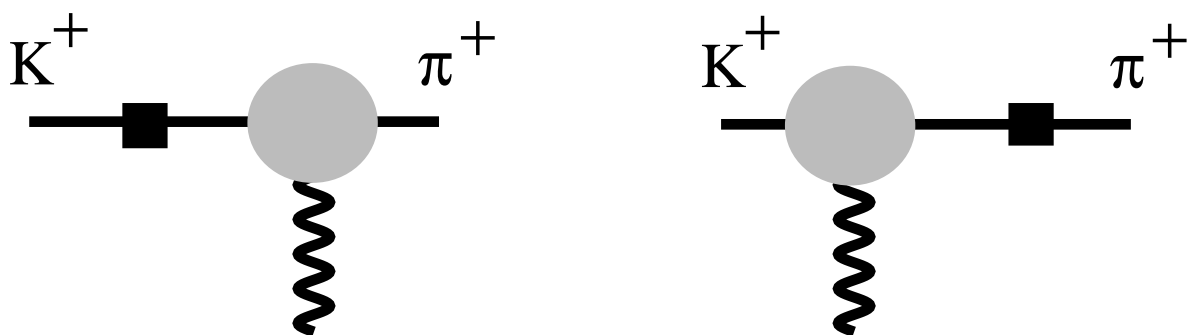


Figure 3(c)

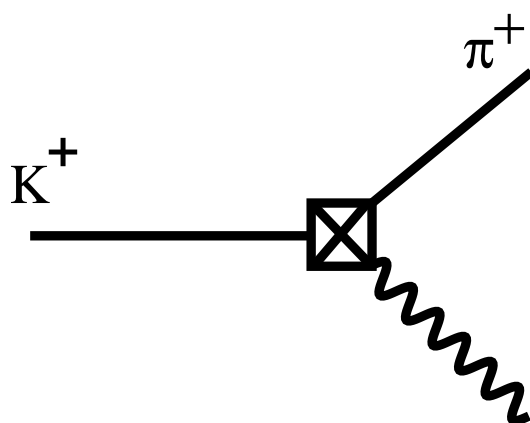


Figure 3(d)

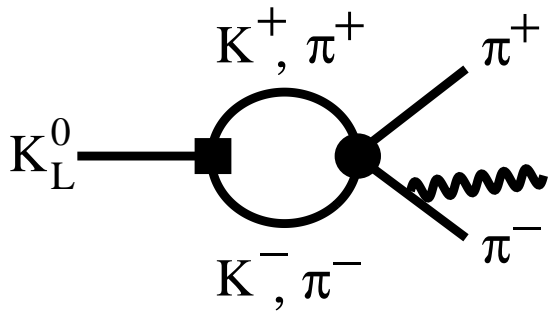
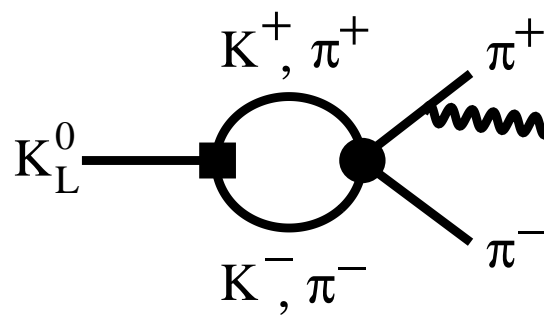
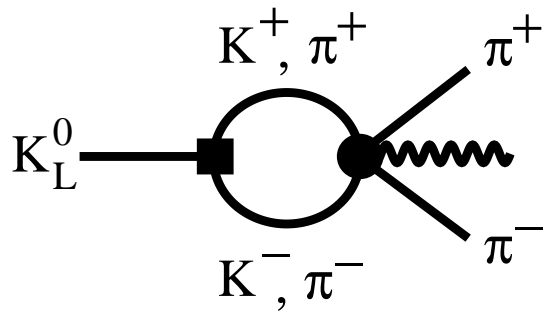
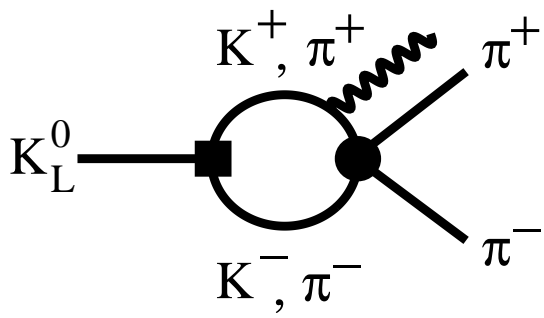
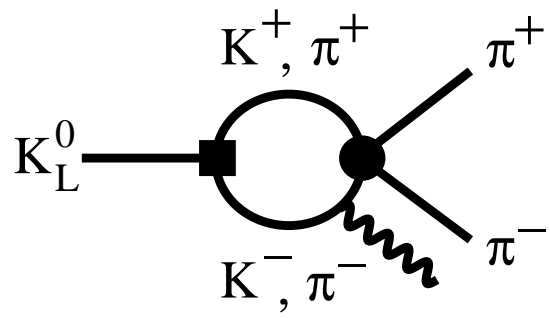
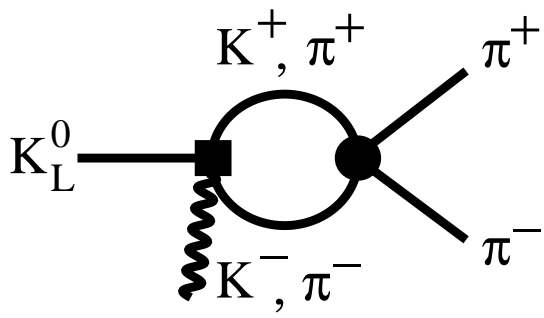
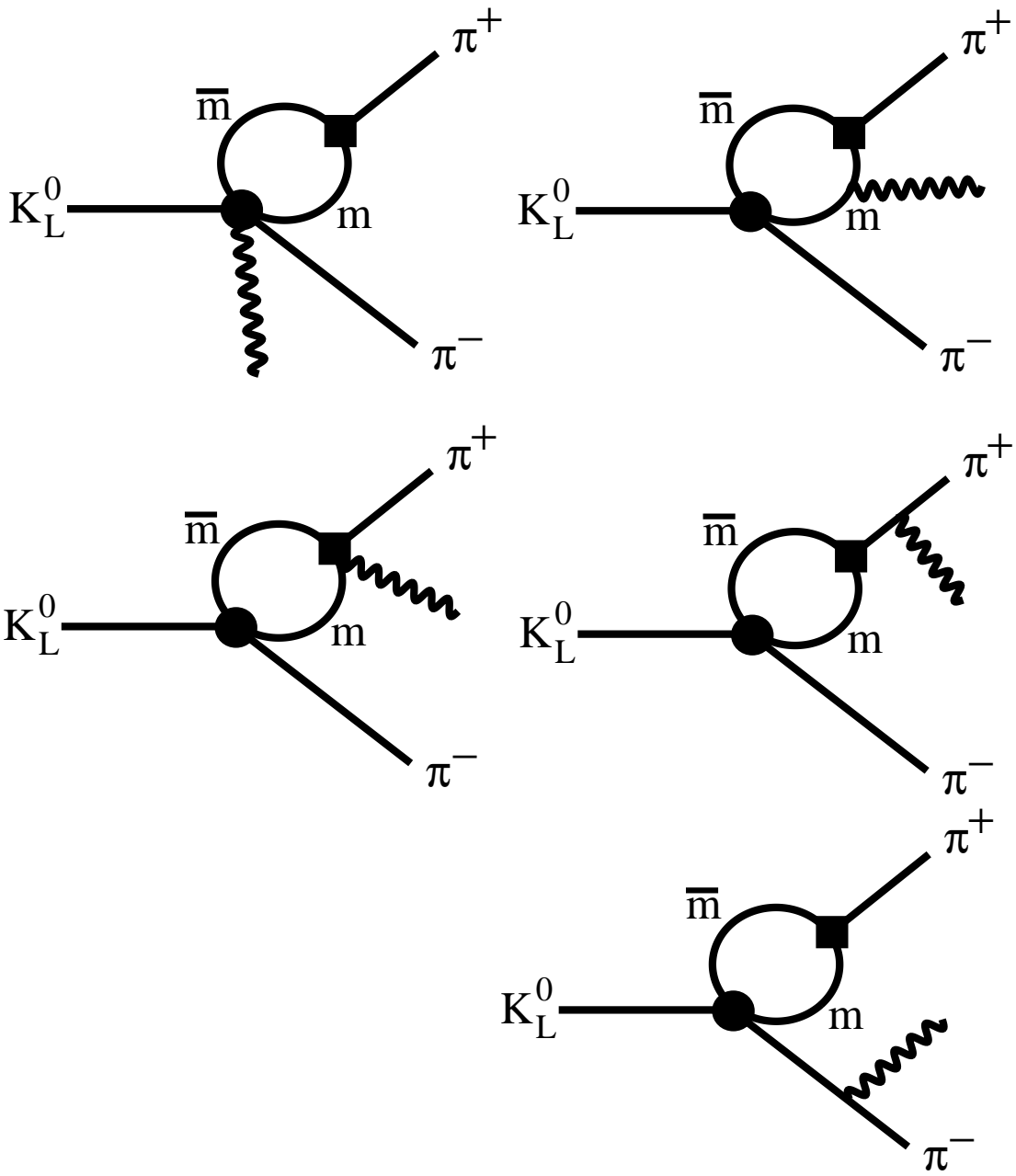
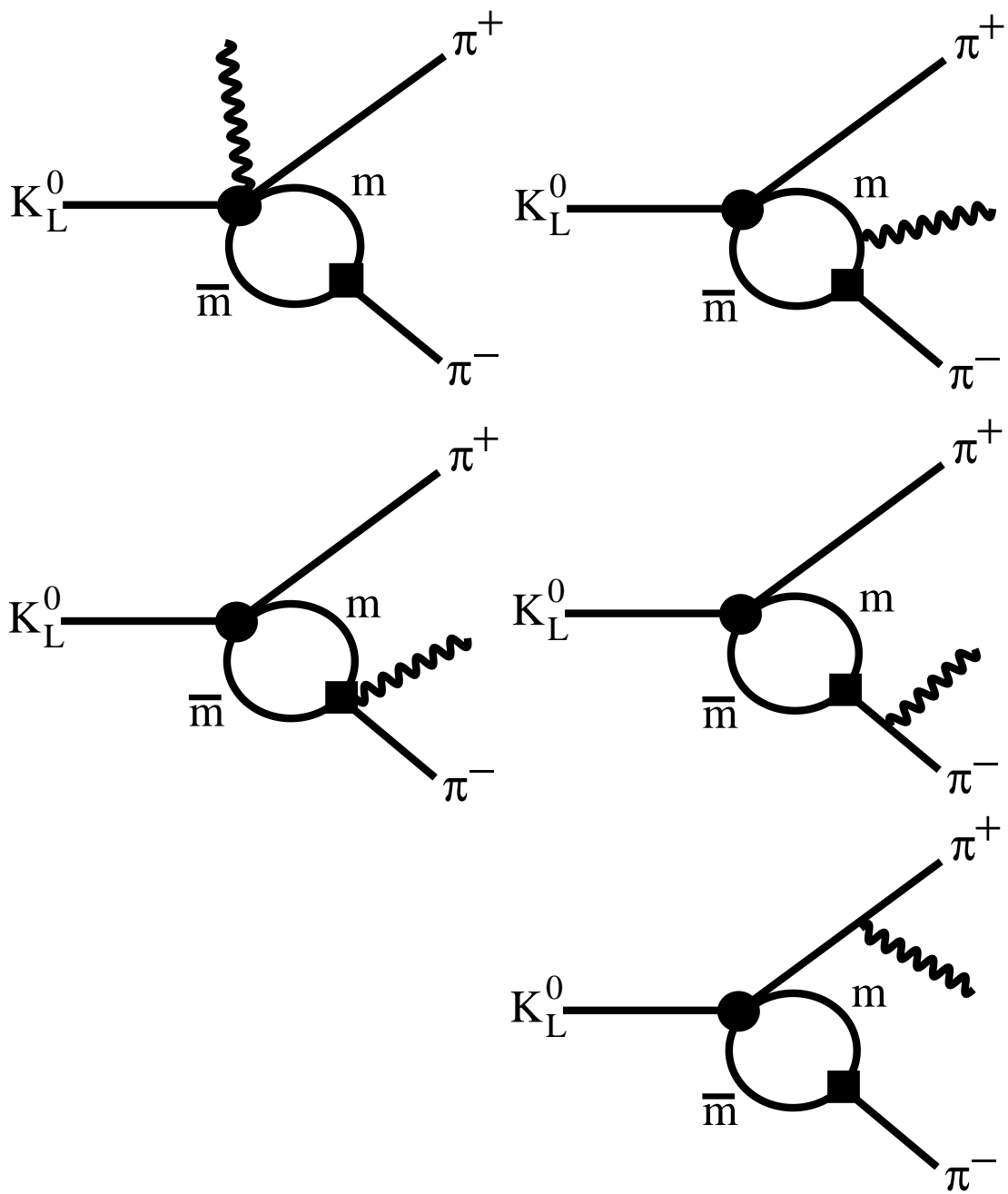


Figure 4(a)



$$(m, \bar{m}) = \begin{pmatrix} (K^+, \pi^0) \\ (\pi^+, K^0) \end{pmatrix}, \begin{pmatrix} (K^+, \eta) \\ (\pi^+, \bar{K}^0) \end{pmatrix}$$

Figure 4(b)



$$\begin{aligned}
 (m, \bar{m}) = & \quad (K^-, \pi^0) \quad , \quad (K^-, \eta) \\
 & \quad (\pi^-, K^0) \quad , \quad (\pi^-, \bar{K}^0)
 \end{aligned}$$

Figure 4(c)

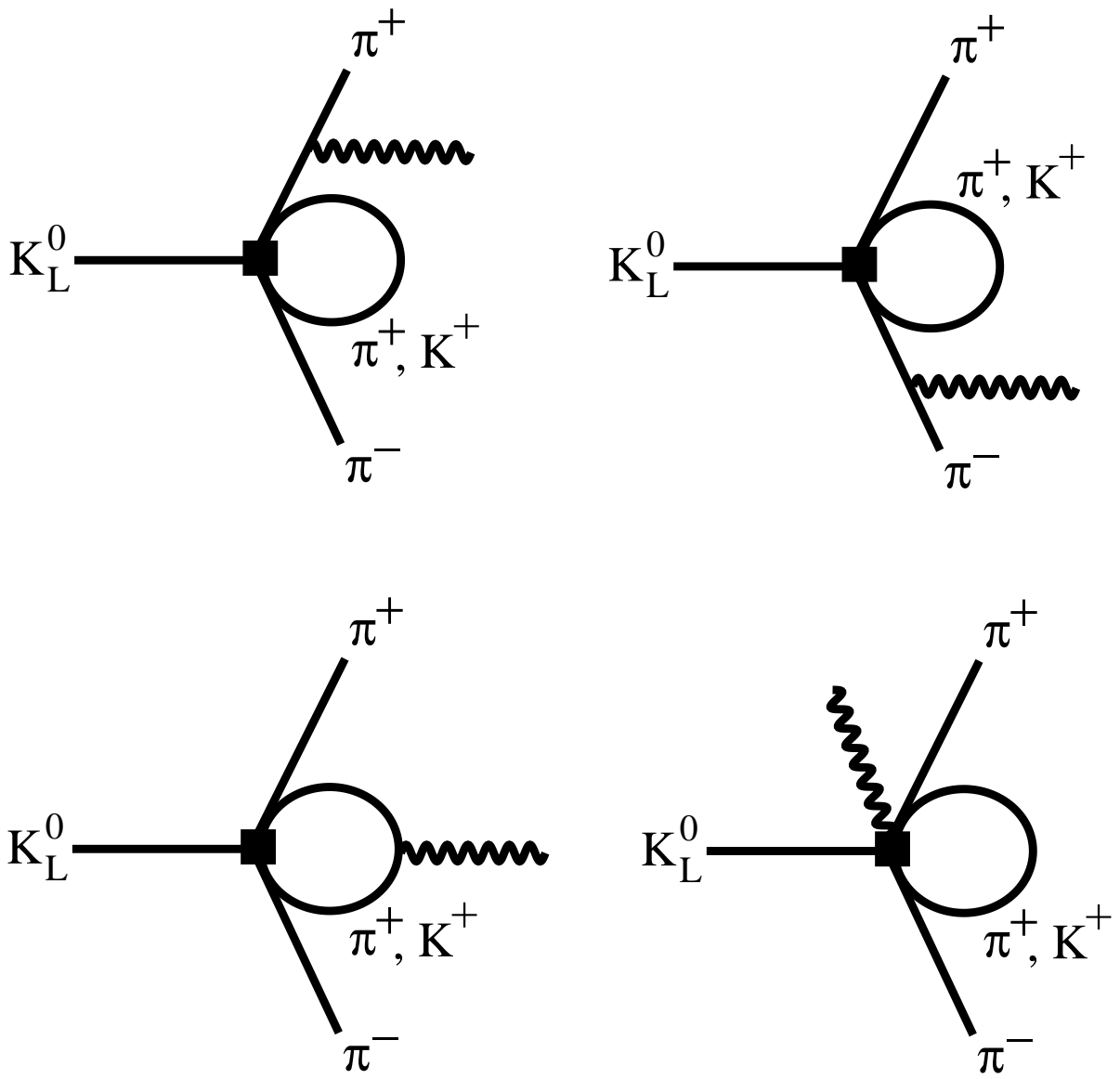


Figure 4(d)

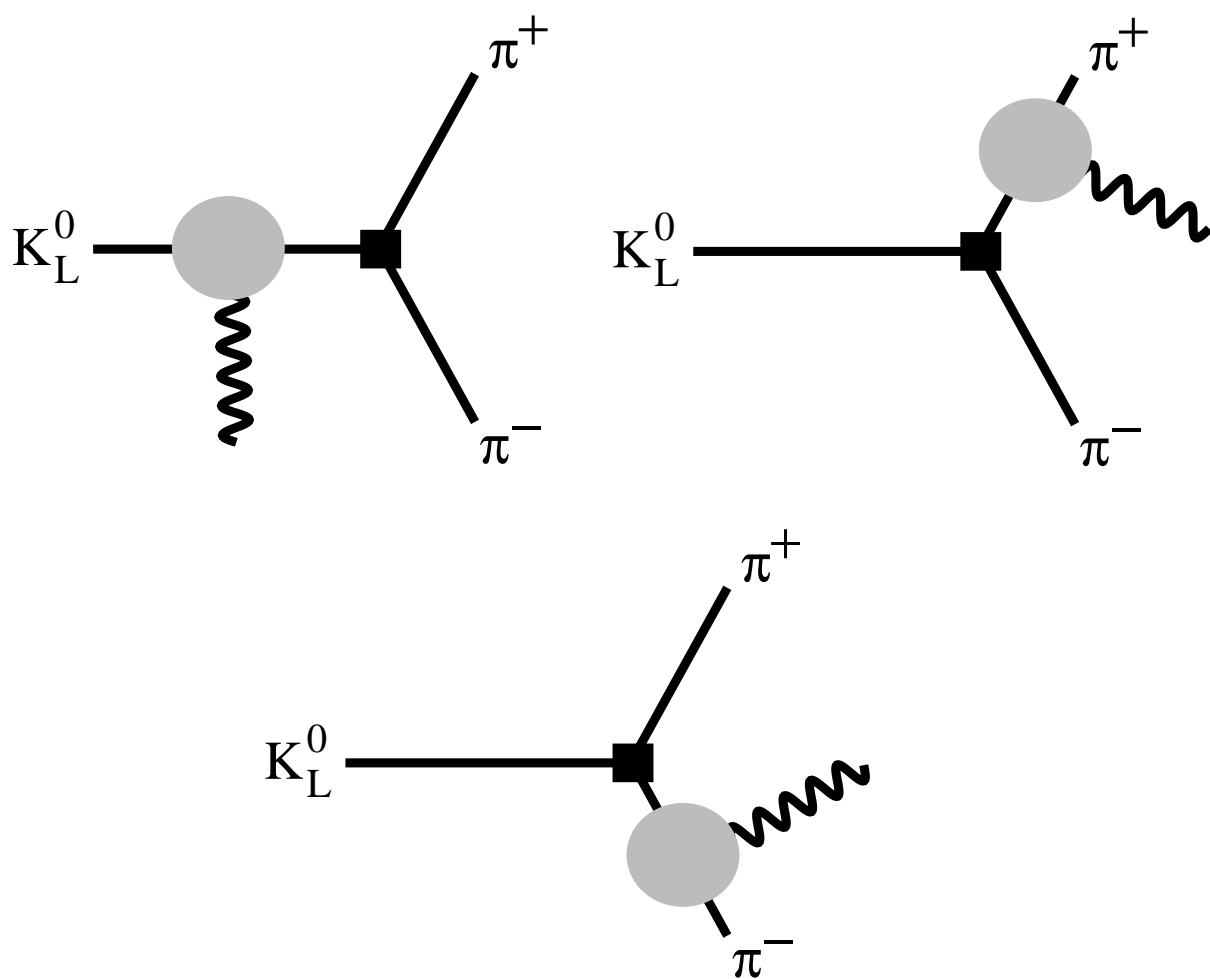


Figure 4(e)

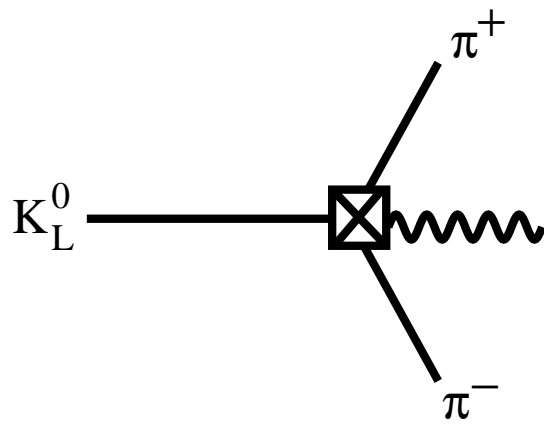


Figure 4(f)

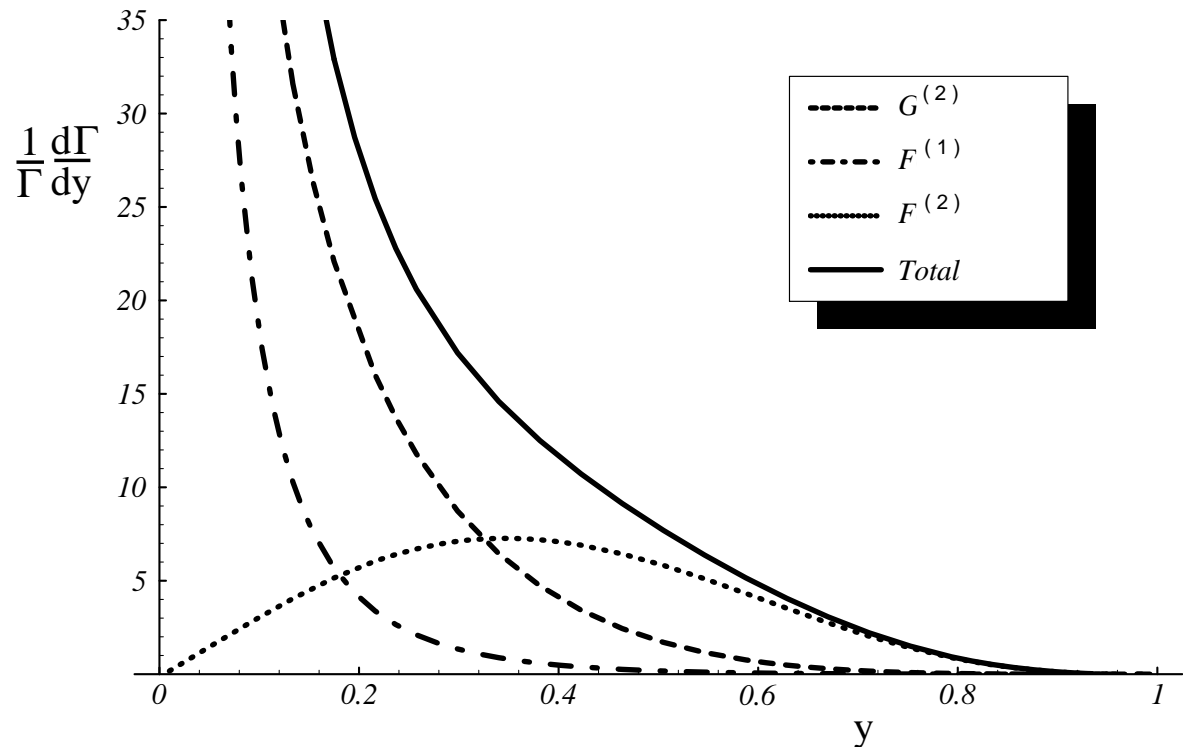


Figure 5